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# Introduction to quotient ordered semirings

*Tapas Kumar Mondal*

**Abstract.** We introduce the notion of pseudoorder on an ordered semiring to define quotient ordered semirings. We also study homomorphism theorems on ordered semirings.

## 1. Introduction

In group theory the notion of normal subgroups play an important role in defining the quotient groups, ideals in defining quotient rings. The notion of congruence takes the place of normal subgroups in a semigroup as well as in semiring to define quotient semigroups and quotient semirings respectively. The situation is not the same for the case of ordered semigroups as given by Kehayopulu and Tsingelis in [7], where, in general given an ordered semigroup $S$ and a congruence relation $\rho$ on $S$, the quotient set $S/\rho$ is not an ordered semigroup. To over come the situation they introduced the concept of pseudoorder on an ordered semigroup $S$ to make $S/\rho$ an ordered semigroup. The notion of semirings was introduced by Vandiver [8] in 1934. It has many applications in idempotent analysis, theoretical computer science, information sciences, ordered semirings etc. Recently, semirings order in semirings was studied by Han, Kim and Neggers [4]. The quotient structure of semirings one can find in [3], [5]. It is well known that each homomorphic image $(\phi(S), +, \cdot)$ of a semiring $(S, +.\cdot)$ without order is isomorphic to the congruence class semiring $(S/\kappa, +, \cdot)$ with respect to the congruence $\kappa = \phi^{-1} o \phi$ determined by $\phi$, in other words, any onto homomorphism $\phi$ from a semiring $(S, +, \cdot)$ to another semiring $(T, +, \cdot)$ without ordered is entirely determined by the congruence $\kappa = \phi^{-1} o \phi$. But for a given ordered semiring $S$, and a congruence relation $\rho$ on $S$, the quotient set $S/\rho$ is not an ordered semiring, in general. So to find all homomorphisms of an ordered semiring to another ordered semiring, we need such relations which are, somehow, 'greater' than congruences. This motivates us to introduce the notion of pseudoorders.

In this paper we study ordered semirings and introduce quotient ordered semirings. In section 2, we have some preliminaries and prerequisites we need to study the underlying semirings. In section 3, we start with an example of an ordered semiring $S$, and a congruence $\rho$ on $S$ such that the quotient set $S/\rho$ is not an ordered semiring. We introduce the notion of pseudoorder $\rho$ on an ordered semiring $S$, the symmetric opening $\overline{\rho} = \rho \cap \rho^{-1}$ of $\rho$ found to be a congruence

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